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Collapsible impact energy absorbers: an overview

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Abstract

This paper reviews the common shapes of collapsible energy absorbers and the different modes of deformation of the most common ones. Common shapes include circular tubes, square tubes, frusta, struts, honeycombs, and sandwich plates. Common modes of deformation for circular tubes include axial crushing, lateral indentation, lateral flattening, inversion and splitting. Non-collapsible systems, such as lead extrusions or tube expansions, are considered to be beyond the scope of this review.

Keywords: Energy absorber; Thin-tubes; Buckling; Crushing; Progressive collapsing

1. Introduction

The general public is becoming increasingly aware of the safe design of components and systems with the objective of minimizing human suffering as well as the financial burdens on society. There seems to be a consensus that much more can be done to lessen the potential dangers of impact accidents.

Impacting events remind us of many types of tragedies like traffic accidents, natural collisions, and earthquakes. The scales of car, airplane and ship collisions are different than the collisions that occur naturally like in earthquakes and falling rocks from high mountains onto roads.

It is interesting to observe that thin-walled members like plates, shells, tubes, stiffeners and stiffened sandwich panels used in automobile bodies, aircraft fuselages

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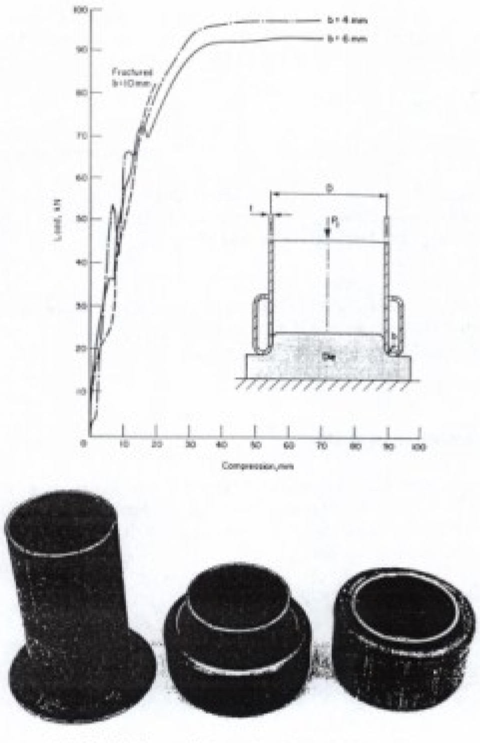

Fig. 3. Inside-out tube inversion due to axial load [47].

tubes known as inverbucktube. His test results show up to a 50% increase in absorbed energy when compared to uniform thickness tube inversion.

4.2. *Tube splitting*

Splitting mode of deformation is a special case of tube inversion where the die radius is large enough to cause splitting instead of inversion [37] (see Fig. 4).

In 1972, Ezra and Fay [80] identified the combined modes of axial splitting and subsequent curling of the split ends of the tubes as an efficient means of energy dissipation in the absorber. The absorbed energy is dissipated in tearing of the metal

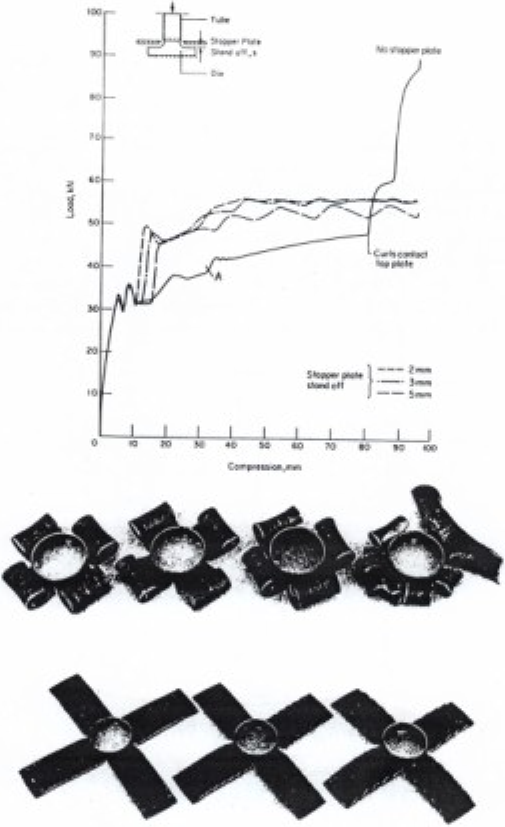

Fig. 4. Tube splitting due to axial loading [47].

of the tube into strips. Stronge et al. [40] examined the splitting of square section tubes, where two modes of splitting were identified, with and without inversion. Tube splitting was investigated by other researchers including Reddy and Reid [81], Atkins [82] and Lu et al. [83].

4.3. Lateral indentation of tubes

Watson et al. [84] studied local loading of tubes, which is a typical example of an automobile bumper under the action of point load, and reported a method of energy calculation which provided reasonable agreement with experimental data. Accordingly, the failure mode started as local denting followed by global bending collapse (see Fig. 5). Johnson and Walton [1,2] investigated experimentally the load–deflection curves of 10 different car bumpers of common passenger cars. The investigation includes local penetration of the bumper as a simply-supported beam subjected to a central load. Their results showed that the investigated bumpers can withstand a maximum impact velocity of 2.8 m/s (10 km/h).

Two-points loading (line loading along the tube length) was also examined by Reid and Bell [86], Carney and Pothen [15], and Gupta and Sinha [87]. Although this type of loading (point load) is very common in practical life, such as collisions between offshore structures and supply boats, no interest is shown in this mechanism

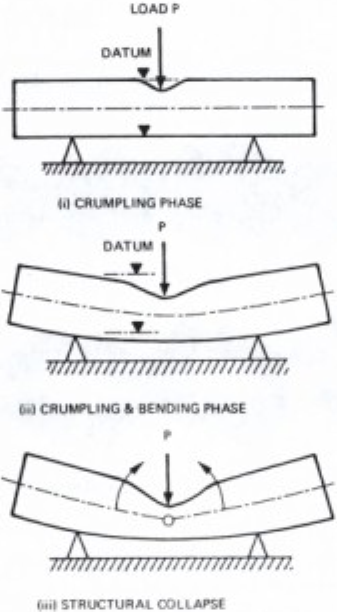

Fig. 5. Lateral indentation of simply supported tube [85].

of deformation because of the limited amount of material participating in plastic deformation.

4.4. Lateral flattening of tubes

Even though the amount of plastic deformation in this mode is not as global as the axial crushing of tubes, it is still much better than that in lateral indentation (point loading). A simple rigid plastic analysis for the lateral compression of a single tube was given by Deruntz and Hodge [88]. Accordingly the flattening force (P) is given by,

$$P=\frac{4Yt^2L}{\sqrt{3}D(1-(\delta/D)^2)^{1/2}} \tag{7}$$

where L is the tube length, D is the tube diameter and δ is the deflection.

Johnson et al. [33] studied the behavior of crossed layers of thin tubes under lateral load. Reddy and Reid [89] investigated the lateral compression of tubes with side constraints and found that the energy absorbed in a constrained system is three times more than that of the free system. Gupta and Khullar [90] studied the transverse in-plane loading of square and rectangular tubes and reported good agreement between theoretical and experimental results.

The flattening of braced metal tubes between flat plates was examined by several researchers, including Johnson et al. [33], Reid et al. [36], Carney and Veillette [48], Reddy et al. [91] and Carney and Pothen [15]. Reid et al. [36] showed that a significant increase in energy absorption occurs for a bracing angle of 15°. Wu and Carney [92] investigated the initial collapse of braced elliptical tubes and concluded that loading the elliptic tubes along their major axes increases the dissipated energy (see Fig. 6). In another paper, Wu and Carney [93] compared their experimental results with finite element predictions using ABAQUS software.

Reid et al. [35] studied the dynamic crushing of a one-dimensional metal ring system using the structural shock theory to model the plastic wave propagating in a rigid–plastic system. Reid and Reddy [94] and Reid and Bell [95] examined experimentally inertia effects in the one-dimensional metal ring system subjected to high-speed end impact. The rings were welded together using flat plates. Reddy et al. [96] repeated the experimental work for a free-ended system where the rings are placed next to each other.

A system of two concentric rings with a layer of tubes between the rings, oriented such that the tubes and rings have parallel axes, was investigated experimentally by Shrive et al. [97].

Reid et al. [17] described the mode of deformation and the behavior of a tubular ring under static and dynamic loads. Each ring consisted of up to four sections of tube cut at 45° and welded together at four joints. The load–deformation characteristics showed marked increases in energy absorbing capacity over the equivalent free tube.

Johnson and Reid [14] observed that, "energy absorbed by a laterally loaded tube

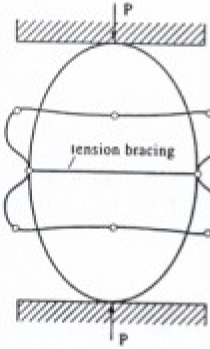

Fig. 6. Tube flattening between two parallel plates [92].

is one order of magnitude less than the axially loaded one". Because of this fact, the crumpling of an axially loaded tube is reviewed in detail below.

5. Axial crushing of circular tubes

The buckling of a circular cylindrical shell under axial load is a classical problem in solid mechanics. From the point of view of energy absorption capacity it was found that circular tubes under axial compression provide one of the best devices. This property perhaps explains why they are the most frequently used components in energy absorber systems [98]. The circular tube proves to be a popular energy absorber because it provides a reasonably constant operating force, which is, in some applications, a prime characteristic of the energy absorber. Further, it has comparatively high energy absorbing capacity and stroke length per unit mass. For example, in comparing lateral compression with axial compression, the axial buckling mode has a specific energy absorption capacity which is approximately 10 times that of the same tube when compressed laterally between flat plates [45]. Moreover, a tube in axial loading can be ensured that all of its material participates in the absorption of energy by plastic work (see Fig. 7). Optimal energy absorption is obtained through progressive plastic buckling which avoids overall elastic buckling.

In the study of the static crushing of structures, material elasticity is unimportant because of the extensive plastic deformation. So the elastic effect is neglected when plastic energy dissipated in the structure is larger than three times the elastic energy of deformation [54]. Also, the initial buckling response of these members is less important, from an energy point of view, than the subsequent post-buckling (yielding) behavior, which is associated with large strains and deflections. This behavior is

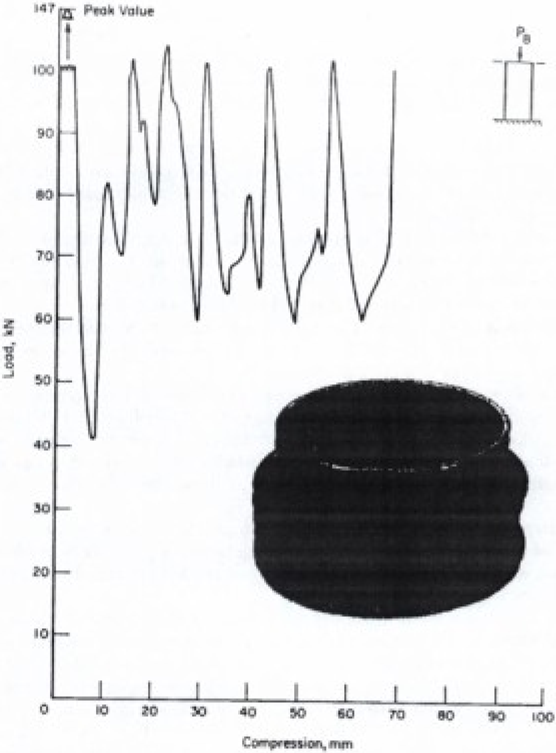

Fig. 7. Axial progressive buckling of metallic energy absorber [47].

often studied assuming the rigid–plastic model since the energy absorbed in the elastic deformation is usually insignificant.

Abramowicz and Jones [99] studied the transition of the axially crushed tubes from the Euler (global) bending mode to the progressive buckling mode at static and dynamic loading conditions. The authors found that the transition point depends on tube length, cross-section, material type, strain-hardening, strain rate and end conditions. They reported that global buckling may or may not coincide with the maximum load-carrying capacity of the column.

Theoretical studies usually ignore dynamic (inertia) effects and treat the problem as a quasi-static case, which is acceptable at low impact velocities.

The behavior of thin-walled tubes (mean diameter (D)/thickness (t)>20) with circular and square cross-sections when subjected to axial loads has been of particular interest since the pioneering work of Pugsley and Macaulay [100] and Alexander [16].

The inextensional model was proposed by Johnson et al. [34], where the authors considered an essentially inextensional mode of deformation and calculated the corresponding mean crushing load. The proposed model provides reasonable estimates of the mean crushing load [45].

Mamalis and Johnson [23] investigated the crumpling of aluminum tubes under quasi-static conditions. Their main objective, among other things, was to determine the experimental details of the failure mode. They fitted empirical equations to their results for both concertina and diamond modes of deformation.

Mamalis et al. [50] repeated the same experimental study using different materials (mild steel) and at elevated strain rates (2.5 m/min). The results were likewise expressed in terms of empirical equations.

Mamalis et al. [101] investigated both theoretically and experimentally the axial crushing of thin PVC tubes with internal grooves. Mamalis et al. [102] also studied the axial crushing of thin bi-material circular tubes. Different materials were used including steel, aluminum and PVC. Mathematical models were developed based on extensible and inextensional analysis of collapse for concertina and diamond modes, respectively.

Wierzbicki et al. [103] proposed a new model for the progressive crushing of circular tubes in an axisymmetric mode of deformation. They introduced an S-shape folding element. The resulting equations were in good agreement with experimental values for $D/t>20$.

An attempt was made to minimize the variation of the crumpling force in axially crushed tubes by Singace and El-Sobky [21]. They used corrugated tubes with different depths to control the formation of plastic hinges and the plastic stretching work.

Little work has been done on the energy dissipation response of stiffened metallic tubes subjected to axial load. It was demonstrated, however, that the insertion of diametrical tension stiffeners [104] or axial stiffeners [105] can dramatically affect the tube load deformation behavior and result in stiffness properties which are directionally sensitive.

5.1. Concertina and diamond modes of deformation

Andrews et al. [106] classified the axial crushing of cylindrical tubes under quasi-static loading into seven different categories, based on experimental observations: (a) sequential concertina; (b) sequential diamond; (c) Euler; (d) concertina and diamond; (e) simultaneous concertina; (f) simultaneous diamond; and (g) tilting of tube axis. Experimental observations of the authors show that thick cylinders (small D/t ratio, $D/t<80$–90) buckle in the concertina (axisymmetric) mode of deformation, whereas thin cylinders (high D/t ratio) buckle in the diamond (non-axisymmetric) mode of deformation. For larger values of D/t where a diamond fold mode of deformation tends to occur, the number of lobes increases with increasing D/t ratio. The diamond mode shows a less specific energy absorption than the concertina mode [106].

Pugsley [107] observed the transition point (from concertina mode to diamond mode) at around D/t=100 while Lee, according to Tvergaard [108], and Mamalis and Johnson [23] observed the point at D/t=30, and 68, respectively. Tvergaard [108] suggested a theoretical model for the prediction of the transition point. His results indicated a transition point at a value of D/t somewhere in the range 50–100. It seems that the transition point depends also on the Y/E (yield strength/modulus of elasticity) ratio. The transition to concertina mode of deformation occurs at a smaller D/t ratio for a larger value of Y/E. For example, only the concertina mode of deformation is observed in very thick-walled PVC (polyvinyl chloride) tubes with $D/t<8$ and Y/E=0.02 [108]. So far, however, no exact analysis has been given which explains why a particular mode of deformation is adopted by a given tube.

5.2. Interactive modes of deformation

Foam-filled circular or square tubes under axial crushing were investigated by many researchers including Reid et al. [109], Abramowicz and Wierzbicki [110] and Reddy and Wall [29], while axial crushing of wood-filled square metal tubes was investigated by Reddy and Al-Hassani [31]. Reid [45] presented load–displacement curves for central transverse loading of tubes filled with sand.

5.3. Average static crushing force

5.3.1. Concertina mode of deformation

Alexander [16] presented a rigid–plastic analysis for the concertina mode of deformation. His model is based on the plastic work required for bending and stretching of an extensible thin cylinder. He gave the following expression for the mean crushing load,

$$P_{av}=6Yt(Dt)^{1/2} \tag{8}$$

where D is the mean tube diameter, t is tube thickness and Y is the yield strength. Eq. (8) can be obtained from Eq. (1) by substituting ϕ=0.

Alexander used the von Mises yield condition for plane-strain conditions to obtain the fully plastic bending moment (M_p) incorporated in his model. Perhaps Eq. (8) has been most frequently used for the axial crushing of tubes in the context of energy absorption criteria. It provides a good prediction for most materials when $D/t<30$ [14].

Abramowicz and Jones [111] improved Alexander's analysis and proposed the average crushing load for the concertina mode of deformation to be in the form,

$$P_{av}=Yt(6(Dt)^{1/2}+3.44t) \tag{9}$$

The predictions using this equation were in good agreement with experiments [47].

Jones and Abramowicz [112] reported a prediction for the crushing force in the form,

$$P_{av} = Yt\,\frac{6\sqrt{Dt}+3.44t}{0.86-0.57\sqrt{t/D}} \tag{10}$$

Eq. (8) is the most famous equation in axial crushing of tubes whereas Eq. (10) gives very good correlation with experimental results. It must be noted, however, that a successful theory needs to take into consideration the large deflection theory and the strain hardening effect [47].

5.3.2. Diamond mode of deformation

Pugsley and Macaulay [100] studied the diamond mode of deformation of thin cylindrical columns (large D/t). In their analysis, energy is assumed to be absorbed by plastic bending and shear of the diamond pattern. They proposed a theoretical estimate of the mean axial load in the form,

$$P_{av} = Yt(10.05t+0.38D) \tag{11}$$

Pugsley [107] proposed a different model for the diamond mode of deformation based on folding of a row of n diamonds. Energy is assumed to dissipate at the plastic hinges during the folding process. Using the same plastic hinge analysis as Alexander [16] they proposed that the average crumpling force has the form,

$$P_{av} = 2.286n^2Yt^2 \tag{12}$$

where n is the number of diamonds formed during crushing. The value of n depends on the D/t ratio. Generally speaking, n increases for large D/t ratios. Thus, Eq. (12) gives a wide range for the average crushing force.

Wierzbicki, as reported by Jones and Abramowicz [112], gives an approximate expression for diamond mode of deformation

$$P_{av} = 18.15Yt^2(D/t)^{1/3} \tag{13}$$

Recently, Jones [113] reviewed the range of validity of quasi-static analysis in thin-walled sections as well as the transition to dynamic plastic buckling and global bending.

6. Dynamic loading

6.1. Impact tests

Dynamic load is defined as that load which is applied at high rate and which is associated with large plastic strains that dominate over the elastic strain. Several theoretical methods for dynamic crushing use the simplified rigid–plastic method of analysis which, often, yields good agreement with experimental results [46].

For low velocity impact, the effect of strain-rate on increasing the yield stress may be included by using a simple factor based on the mean strain-rate in the critical plastic zones. Moreover, inertia effects within the device itself are deemed unimportant and hence the kinetic energy is considered to be converted into plastic work in a quasi-static deformation mode [114].

In terms of the behavior of tubes of various cross-sectional shapes, circular tubes are somewhat unique in having essentially the same mode of deformation in both static and dynamic compression (especially at low velocity impact). Investigations of dynamic compression of circular tubes indicate that the deformation is mainly concentrated at the end of the cylinder that is impulsively loaded [114].

Dynamic plastic buckling can be classified into two types: (a) instantaneous dynamic plastic buckling (which is associated with impact events in which the inertia forces play an important role at high impact velocities); and (b) dynamic progressive buckling where buckling is formed progressively from one end of the tube as the deformation proceeds [115]. This mode develops at low impact velocities when the inertia forces are negligible, and it is essentially the same as for static loading. The transition from progressive mode to instantaneous mode has not been established yet [46].

Ren et al. [116] investigated the dynamic plastic buckling of cylindrical shells made of aluminum alloy, which shows considerable strain hardening with some strain rate effect, and reported that the mode of deformation is affected by the impact velocity (V). They presented simple theoretical analyses based on a rigid linear strain hardening material, and neglecting the stress wave effect as well as the influence of end effects. It was found that when the impact velocity is less than a certain critical value V_{cr1}, the cylinder will exhibit only uniform plastic deformation in both axial and radial directions. Likewise, when the velocity V is greater than V_{cr1} the concertina mode of deformation begins to appear. On the other hand, when the impact velocity exceeds another critical value V_{cr2} (greater than V_{cr1}) the deformation mode will change from concertina to diamond mode of deformation.

6.2. Strain-rate and strain-hardening effects

For any type of energy absorber, a consideration should always be kept in mind, as to the influence of strain-rate on the yielding stress of materials. Postlethwaite and Mills [49] studied the dynamic crushing of metal structures. They observed that the absorbed energy in dynamic impact of tubes would be obtained by a scaling factor, and that both the static and dynamic plastic wavelengths are the same. It was observed that the material strain rate sensitivity exercises an important effect on the response of structures, and should be taken into account for strain-rate sensitive materials [46]. An empirical equation for the dynamic yield stress (Y_d) is given by,

$$Y_d = Y(1+(\dot{\varepsilon}/c)^{1/p}) \tag{14}$$

where $\dot{\varepsilon}$ is the strain rate, c and p are constants to be determined experimentally. Values of c=40.4 s^{-1} and p=5 were obtained experimentally for mild steel [112].

and ship hulls are all characterized by similar thickness-to-width ratios. These structures are subjected to predominantly compressive loads during impact, and they can undergo large deflections that may exceed the associated wall thickness by two orders of magnitude.

The impact of transport vehicles is an unfortunate but common occurrence. It is becoming apparent that, in the future, transport structures will have to be designed to withstand impacts and crashes. The current trend in producing lighter structures puts greater demands on the designer since more aspects of design become critical as the weight is reduced, and working stresses become closer to the ultimate strengths of the material. This is particularly so with light metallic alloys and composite materials which have directional properties. It is expected that the results of composite research will be of specific use to designers of boats, cars, aircraft and other applications.

2. Energy absorbers

During the second half of the last century a great number of impact engineering problems were investigated, especially in the field of the dynamic response of structures in the plastic range. This contributed towards a better understanding of the modes of failure and the energy dissipation patterns during impact in such structures. Such information is important in order to be able to build safer structures and also in evaluating existing ones for specific uses, therefore reducing losses in human and material resources. Application of this field of engineering is now available for use in a wide variety of situations, which include such aspects as crashworthiness of vehicles (cars, lifts, aircraft, ships ...) [1,2], crash barrier design [3], safety of nuclear reactors [4], collision damage to road bridges [5] and offshore structures and oil tankers [6].

Many papers relevant to this field were presented in the 1st International Symposium on Structural Crashworthiness held in Liverpool in September 1983. Since then results of other research and reviews were reported in special issues of the *International Journal of Mechanical Science* (9/10, 1983 and 12, 1993), the *International Journal of Impact Engineering* (founded in 1983), and in books such as *Crashworthiness of Vehicles* by Johnson and Mamalis [7], *Structural Crashworthiness* edited by Jones and Wierzbicki [8], *Structural Impact and Crashworthiness* edited by Davies and Morton [9], *Metal Forming and Impact Mechanics* edited by Reid [10], *Structural Crashworthiness and Failure* edited by Wierzbicki and Jones [11], *Structural Impact* by Jones [12], and *Structural Crashworthiness and Failure*, edited by Jones and Wierzbicki [13].

2.1. Definition

An energy absorber is a system that converts, totally or partially, kinetic energy into another form of energy. Energy converted is either reversible, like pressure energy in compressible fluids and elastic strain energy in solids, or irreversible, like

Average strain rate for the concertina mode of deformation was estimated by Abramowicz and Jones [111] to be,

$$\dot{\varepsilon}=\frac{V}{2D\{0.86-0.568\sqrt{t/D}\}} \tag{15}$$

where V is the impact velocity, and

$$\dot{\varepsilon}=\frac{0.74V}{D} \tag{16}$$

for the diamond mode of deformation.

The effect of strain hardening appears to be small unless the material has very strong hardening properties. According to experimental work on mild steel and aluminum, the flow stress is roughly 10% larger than the static yield stress. Thus, any correction to an analysis may be even smaller than 10%, since the maximum stress is reached only in some part of the structure under investigation [12].

6.3. Inertia effect

The inertia effect in impact energy absorbers has been investigated by many researchers including Reid and Reddy [94] and Harrigan et al. [117]. Reid and Reddy examined experimentally the inertia effect in one-dimensional arrays of laterally compressed metal tubes and rings. It was observed in their study that the plastic deformation is concentrated at the rings in the impacted side of the array. Harrigan et al. [117] studied the inertia effect both experimentally and numerically in metal tube inversion and axial crushing of aluminum honeycombs. The inertia effect tends to reduce the crushing force in tube inversion as the inversion velocity increases, and this is attributed to the mechanism of deformation in tube inversion. The inverted mass helps to pull out the material at the die and hence reduce the crushing force.

In summary, the inertia effect can be classified into two categories:

1. structural shapes (or devices) where inertia plays no role. A typical example would be axial crushing of tubes at low impact speed; and
2. structural shapes where inertia does play a significant role either to minimize the crushing load, as in tube inversion [117], or to maximize it as in cubic rod cells [5].

Other issues such as: scaling [82,118], rotary inertia effect [46], brittle fracture [12], material sensitivity [119] and imperfected column [120] were discussed in the literature.

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plastic deformation energy. Energy dissipated in plastic deformation of metallic energy absorbers is the absorbing system reviewed in this paper.

When designing a collapsible energy absorber, one aims at absorbing the majority of the kinetic energy of impact within the device itself in an irreversible manner, thus ensuring that human injuries and equipment damages are minimal. The conversion of the kinetic energy into plastic deformation depends, among other factors, on the magnitude and method of application of loads, transmission rates, deformation or displacement patterns and material properties [14].

The components of deformable energy absorbers include such items as steel drums [15], circular tubes [16], tubular rings [17], square tubes [18–20], corrugated tubes [21], multicorner columns [22], frusta [23], struts [24], honeycomb cells [25], sandwich plates [26] and some other special shapes such as stepped circular thin-walled tubes [27] and top-hat thin-walled sections [28].

These elements were used when filled with liquids, foam [29,30], wood shavings [31] and sand. These elements can be arranged in a variety of geometries. Some of the most well-known arrangements include, axial crushing of tubes [32], lateral crushing of tubes [33–36], tube inversion [37,38], tube nosing [39] and tube splitting [40]. Many researchers investigated the crushability and absorption rate of some classical materials. These include wood [41,42] and concrete [43].

Johnson and Reid in two review papers [14,44] identified the dominant modes of deformation of simple structural elements. In 1985, Reid [45] reviewed the progress in metallic energy absorbers from 1978 to 1985. Later on, Jones [46] published a literature overview article on the dynamic plastic behavior of structures in which he cited 194 references, the majority of which were published after 1978. In 1993, Reid [47] reviewed plastic deformation of axially compressed energy absorbers.

Each energy absorber system has its own characteristics and special features which one needs to be familiar with in order to be able to understand how metallic structures respond to impulsive loads. Because of the extreme complexities of collapse mechanisms, some of these performance characteristics were determined only through experimental procedures [48]. Consequently, the resulting empirical relations are confined to limited applications.

The study of deformation in energy absorbers accounts for geometrical changes, and interaction between various modes of deformation such as the concertina (axisymmetric) mode of collapse and the diamond (non-axisymmetric) mode of collapse, for axially loaded tubes, as well as strain hardening and strain rate effects.

3. Absorber shape

There are numerous types of collapsible impact energy absorbers that are cited in the open literature. In this section the most common shapes are reported.

3.1. Tubes

Thin tubes represent the most widespread shape of collapsible impact energy absorbers, owing to their high frequency of occurrence as structural elements. Details of the crushing modes of circular tubes under static and dynamic loading conditions are given in Sections 4, 5 and 6.

3.2. Frusta

Frusta (truncated circular cones) have wide ranges of applications. The occurrence of frusta as structural members has drawn some attention, especially due to their stable plastic behavior when crushed axially. The literature on this topic, however, is generally meager [23].

The frustum was first studied by Postlethwaite and Mills [49]. They used Alexander's method (extensible collapse analysis) for rigid–perfectly plastic material cones. They reported the mean crushing force (P_{av}) for external collapse as

$$P_{av}=6Yt^{3/2}\sqrt{d+2x\sin(\phi)}+5.69Y\,t^2\tan(\phi) \tag{1}$$

where Y is the yield strength, t is the frustum thickness, x is the deformation, d is the small diameter of the frustum and ϕ is the semi-apical angle of the frustum.

Mamalis and Johnson [23] experimentally investigated the quasi-static crumpling of aluminum tubes and frusta under quasi-static compression. Their main objective was, among other things, to determine the experimental details of the failure modes of frusta. It was observed that load–deflection curves of the frusta are more regular than those of cylinders. Also, post-buckling load increases in a parabolic manner with increases in wall thickness, and, as expected, post-buckling load decreases with an increase in semi-apical angle. It was observed that thin frusta deformed into a diamond shape whereas thick ones deformed into axisymmetric rings. The authors fitted empirical equations to their results for both concertina and diamond modes of deformation.

Mamalis et al. [50] repeated the same experimental study using low-carbon steel and at elevated strain rates (2.5 m/min). It was observed that the initial axisymmetric bellows changes into non-symmetric diamond shapes and the number of lobes of the diamond shape increased as the ratio of the mean diameter/thickness increased.

Mamalis et al. [51] proposed an extensible theoretical model to predict the plastically dissipated energy and the mean post-buckling load for axially crumpled thin walled circular cones and frusta for the concertina mode of deformation. The theoretical model was based on a consideration of the plastic work dissipated in plastic hinges and in stretching of material between them without considering their interaction. The model gave the average crushing load in the form,

$$P_{av}=6Yt^{3/2}(\sqrt{d}+0.95\sqrt{t}\tan(\phi)) \tag{2}$$

Predicted average crushing loads were in fair agreement with experimental results.

Mamalis et al. [52] developed a theoretical model to predict the mean crushing load for axially loaded circular cones and frusta deformed into the diamond mode of deformation. The model was based on the inextensional model developed by Johnson et al. [34].

Mamalis and co-workers [53] improved the analytical model for the concertina mode of deformation by making it capable of predicting the deformation history of thin-wall tubes and frusta. They obtained long and tedious equations for internally and externally formed convolutions, which were in fair agreement with the experimental curves.

Mamalis and his group [54] studied the axial crushing of thin PVC frusta of square cross-sections. A theoretical model for prediction of the average crushing force was developed on the basis of an inextensional folding mechanism of the diamond mode of deformation. Good correlation between the experimental and analytical results was shown.

Mamalis et al. investigated the axial collapse of composite tubes [55] and frusta of square [56] and circular [57] sections and developed an analytical model of the crushing stages based on actual experimental observation.

Alghamdi [58] introduced two innovative modes of deformation for frusta. The first one is direct inversion (see Fig. 1) and the other one is outward flattening. Using the ABAQUS finite element program, Aljawi and Alghamdi [59] modeled the collapse of frusta when inverted. Good agreement was obtained between experimental results and theoretical predictions. Aljawi and Alghamdi [60] further investigated the details of the inversion of frusta when crushed axially. Alghamdi et al. [61] presented the details of crushing, between two parallel plates, of spun frusta, and classified the deformation modes into three modes: (1) outward flattening; (2) inward inversion and outward flattening; and (3) extensible crumpling. They reported

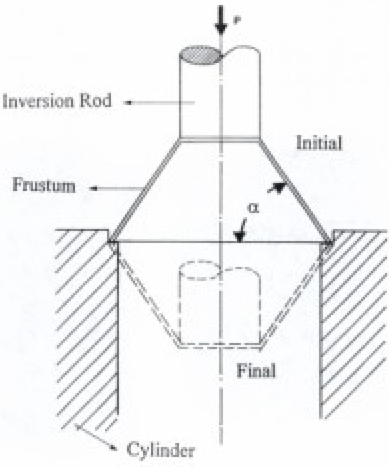

Fig. 1. Direct inward inversion of frusta [60].

that their predictions using ABAQUS were in good agreement with experimental results.

3.3. Multicorner columns

Wierzbicki and Abramowicz [22] analyzed the crushing of thin-walled multicorner structures made from plate elements, by considering stationary plastic hinges and narrow toroidal regions of circumferential stretching and bending which travel through the structure. As a special case of the multicorner column, the mean crushing load (P_{av}) for the symmetrical collapse mode of a square tube (see Fig. 2) made of rigid–plastic material takes the form,

$$P_{av}=9.56\,Yt^{5/3}C^{1/3} \tag{3}$$

where C is the width of the square tube and t is the column thickness.

Abramowicz and Jones [62] predicted the average static crushing force for asymmetric collapse of a square tube to be,

$$P_{av}=Yt(9.69C^{1/3}t^{2/3}+0.84C^{2/3}t^{1/3}+0.58t^{2}) \tag{4}$$

In 1989, Abramowicz and Wierzbicki [63] improved their previous model by assuming an arbitrary angle between the adjacent plates of the structure. Accordingly, the average crushing load (P_{av}) for the square tube is,

$$P_{av}=13.06\,Yt^{5/3}C^{1/3} \tag{5}$$

Eq. (5) takes into account the average flow stress and it was found to be in good agreement with experimental results.

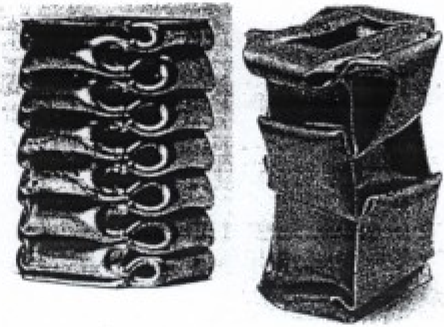

Fig. 2. Axially crushed square tubes [47].

3.4. Struts

Postlethwaite and Mills [49] were among the first researchers to use simple struts made of mild steel as impact energy absorbers. A simple strut with an initial imperfection (deflection) was also used as an energy absorber by Grzebieta and Murray [64]. Because of the limited zone of plastic deformation, i.e. one plastic hinge, the absorbed energy in static tests was minimal. The initial imperfection amplitude was used to control the absorbed energy, the maximum force, and hence the deceleration rate of the impacting mass. Harris and Adams [24] investigated both theoretically and experimentally the crushing behavior of structures made by bonded and spot-welded lap joints. Mild steel and aluminum tubular specimens made by a single lap joint were tested. Grzebieta and Murray [65] repeated their previous work but for a dynamic loading condition. Recently, Reid and Sicking [3] have studied large plastic deformations of sequential kinking in guardrail terminals. A non-linear, large deformation finite element package (LS-DYNA) was used to model the deformation sequence. Drazetic et al. [66] modeled the crushing of S-frames using finite element analysis.

3.5. Sandwich plates

The energy absorbing mechanisms of impacting events in sandwich structures made of composites are quite unusual. Sandwich structures are widely used in a variety of applications. They are common in transport vehicles, such as aircraft, trains, cars and boats and also in the construction industry for buildings.

A sandwich structure is constructed from a core with a skin attached to each side, and is analogous to an I beam, where the core represents the web and the skin the flanges. Since there is a wide range of materials that can be considered for the skin and core, there exists considerable flexibility for the designer, presenting an overwhelming combination of materials for the construction of sandwich structures. This is one reason why it is necessary to develop a validated theory for the design of sandwich constructions. This theory could then be used with confidence by designers to predict the behavior of a wide range of sandwich constructions and thus select the best combination for a given design problem. With increasing interest in the design of structures to withstand impact, considerations of energy absorption are becoming important.

The impact and energy absorption of sandwich structures has drawn the interest of many workers. Thus Gilkie and Sundararaj [67] investigated the effect of: (1) laminate thickness; (2) core thickness; (3) facing thickness; and (4) support span on the impact strength of the sandwich. Some tests were also conducted on glass fiber laminates to provide a comparison. They reported that sandwich panels were substantially more resistant to impact failure than simple laminates. Results show that the impact strength of the front skin is independent of the core thickness but the rear skin impact strength increases as the core thickness increases.

Raschbichler [68] described briefly some tests carried out on a prototype vehicle chassis constructed of sandwich structures. He concluded that the advantage of sandwich panels for energy absorption is that the deformation remains local to the impact area and that the damage is not passed on to remote parts of the structure.

Rhodes [69] conducted impact tests by firing aluminum spheres 12.7 mm in diameter at velocities from 16 to 67 m/s using a gas gun. He found that low energy impacts, which cause no visible damage, can initiate failures in stressed sandwich structures, and it may be possible to use a fracture mechanics approach to predict the residual tensile strength of sandwich structures.

Sponberg [70] described briefly some impact tests carried out on sandwich structures with five different constructions. No details of the impact test method were given however; in particular no description of the shape of the indenter was given.

Worrall [71] investigated the low velocity (3–7.5 m/s) impact of sandwich panels with aluminum alloy honeycomb cores with mainly glass fiber reinforced epoxy resin or polyester resin skins. He compared the experimental results with one and two degrees of freedom mass spring models.

Goldsmith and Sackman [72] presented the details of an experimental investigation concerned with static and dynamic loading of aluminum honeycomb covered by aluminum and non-metallic plates. It was concluded that both the static and dynamic responses are similar to each other with the dynamic response 20–50% more than the static value.

Wray [73] of UMIST aimed to use sandwich structures in the offshore industry. Three failure modes were observed in the experiments on the quasi-static loading of sandwich structures, local indentation at the loading point, core shear failure and skin compressive failure. No tensile skin failures were observed. Some of the core shear failures were also accompanied by delamination at the skin to core interface. The thick skin sandwich beams with transverse core ribbon direction appeared to crush along the whole length of the panel due to severe shearing of the core. It was concluded that the sandwich structures with thin skins (0.67 mm) and thick cores (19.5 mm) tended to fail by the indentation mode at short spans. Typically compressive skin failures tended to occur in sandwich structures with long spans (430 mm) and thin skins (0.67 mm). The failure modes observed in the impact tests were similar to those for the quasi-static tests.

It was further reported that indentation tests on sandwich constructions supported on a rigid block showed that the core fails (e.g. honeycomb cores) at first peak load but the skin remains intact. Records made clear that the failure load was increased compared with the quasi-static indentation tests.

3.6. Honeycomb cells

Wierzbicki [25] developed a model to predict the average axial force of metal honeycombs (hexagonal structure) based on plastic work dissipated in bending and extension. The mean crushing force (P_{av}) is given as,

$$P_{av}=8.61\sigma_0 t^{5/3}C^{1/3} \tag{6}$$

where σ_0 is the average flow strength. Wu and Jiang [74] investigated experimentally static and dynamic behavior of aluminium honeycomb and reported an increase in the crushing force of up to 74% in the dynamic case when compared to the quasi-static case. In contrast, Zhao and Gary [75] gave the percentage difference as being up to 40% between dynamic and static cases.

3.7. Other shapes

Other shapes that were utilized in energy absorption work include:

1. W-frame made of four rods connected by three elbows [14];
2. polygonal cross-section cylinders subjected to lateral [14] and axial [76] loads;
3. wave-shape guard fence made of bent pipes [14];
4. cubic rod cell [5];
5. three-dimensional tubular system [5];
6. inversion of spherical shells [77] and axial crushing between rigid plates [78];
7. symmetric stepped circular thin-walled tube [27]; and
8. single-hat and double-hat thin walled sections [28].

4. Tube deformation modes

Because of their high frequency of occurrence as structural elements, tubes are considered to be the most common shape and probably the oldest shape utilized in energy absorption. Plastic energy can be dissipated in thin metallic tubes in several modes of deformation, including:

1. inversion;
2. splitting;
3. lateral indentation;
4. lateral flattening; and
5. axial crushing.

4.1. Tube inversion

One of the interesting energy absorber columns is the tube inversion or invertube that basically involves the turning inside out or outside in of a thin circular tube made of ductile material, as shown in Fig. 3. This method was introduced by General Motors in 1969 as reported by Al-Hassani et al. [37]. The main advantage of this mode of deformation is the constant inversion load that can be obtained for a uniform tube. Characteristics of tube inversion were investigated by Al-Hassani et al. [37], Kinkead [79], Chirwa [38] and Reid [47]. Note that tube inversion is limited by die radius. If the die radius is small, progressive buckling of the tube will result, and if the radius is larger than some limiting value, tube splitting will result. Chirwa [38] investigated both experimentally and analytically the inversion of tapered thin-walled