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Approximating fixed points of non-self nonexpansive mappings in Banach spaces

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Abstract

Suppose K is a nonempty closed convex nonexpansive retract of a real uniformly convex Banach space E with P as a nonexpansive retraction. Let $T : K \to E$ be a nonexpansive non-self map with $F(T) := \{x \in K : Tx = x\} \neq \emptyset$. Suppose $\{x_n\}$ is generated iteratively by

$$x_1 \in K, \ x_{n+1} = P((1-\alpha_n)x_n + \alpha_n TP[(1-\beta_n)x_n + \beta_n Tx_n]),$$

$n \geqslant 1$, where $\{\alpha_n\}$ and $\{\beta_n\}$ are real sequences in $[\varepsilon, 1-\varepsilon]$ for some $\varepsilon \in (0, 1)$. (1) If the dual E^* of E has the Kadec–Klee property, then weak convergence of $\{x_n\}$ to some $x^* \in F(T)$ is proved; (2) If T satisfies condition (A), then strong convergence of $\{x_n\}$ to some $x^* \in F(T)$ is obtained.

MSC: 47H09; 47J25

Keywords: Nonexpansive non-self map; Demiclosed map; Kadec–Klee property

1. Introduction

Let K be a nonempty subset of a real normed linear space E. Let T be a self-mapping of K. Then T is said to be *nonexpansive* if

$$\|Tx - Ty\| \leqslant \|x - y\| \tag{1.1}$$

for all $x, \ y \in K$.

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