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Common fixed points of almost generalized contractive mappings in ordered metric spaces

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A B S T R A C T

The existence theorems of common fixed points for two weakly increasing mappings satisfying an almost generalized contractive condition in ordered metric spaces are proved. Some comparative example are constructed which illustrate the values of the obtained results in comparison to some of the existing ones in literature.

1. Introduction and preliminaries

In 1968, Kannan [15] proved a fixed point theorem for a map satisfying a contractive condition that did not require continuity at each point. This paper was a genesis for a multitude of fixed point papers over the next two decades. On the other hand, Sessa [19] introduced the notion of weakly commuting maps, which are generalization of commuting maps, while Jungck [13] generalized the notion of weak commutativity by introducing compatible maps and then weakly compatible maps [14].

Definition 1.1. Let (X,d) be a metric space. A map $g : X \to X$ is called an almost contraction with respect to a mapping $f : X \to X$ if there exist a constant $\delta \in]0,1[$ and some $L \geqslant 0$ such that

$$d(gx,gy) \leqslant \delta d(fx,fy) + Ld(fy,gx),$$

for all x, $y \in X$.

If we choose $f = I_X$, I_X is the identity map on X, we obtain the definition of almost contraction, the concept introduced by Berinde ([4,5]). This concept by Berinde in [4] was called as 'weak contraction', but in [5], Berinde renamed it as 'almost contraction' which is appropriate. Berinde [4] proved some fixed point theorems for almost contractions in complete metric spaces.Then many authors have studied this problematic and obtained significance results ([3,6–12,16,17]).

It was shown in [4] that any strict contraction, the Kannan [15] and Zamfirescu [20] mappings, as well as a large class of quasi-contractions, are all almost contractions.

Let g and f be two selfmaps of a metric space (X,d). g is said to be f-contraction if there exists $k \in [0,1)$ such that $d(gx,gy) \leqslant kd(fx,fy)$ for all $x,y \in E$.

In 2006, Al-Thagafi and Shahzad [1] proved the following theorem which is a generalization of many known results.

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